VOLUME AS A FUNCTION OF PRICE - AN ALTERNATE APPROACH

ARVIND HARIHARAN AND HEMANTH HARIHARAN

1. Introduction

The energy component of Full Requirements (FR) deals is usually a short position in a variable-volume swap, where the volumetric component is typically valued using an actuarial approach. This paper explores an alternative approach where the volume can be approximated as a function of price to come up with a semi-analytical formulation for the arbitrage-free price and Greeks for hedging the swap. This valuation approach allows the exotic swap to be booked as a standard derivative whose value depends only on traded parameters.

2. Valuation

Let's define the following variables:

1. L - Load (Volume)
2. P - Floating Price
3. k - Fixed Contract Price
4. t - Time

In a typical FR deal, we sell L MWhs of power at the fixed contract price k while we buy power at the floating market P price. Setting the expected value of the payoff of this deal to 0, we get

$$E\left[\sum_{m=1}^{M}(k-P_m)L_m\right]=0 \tag{2.1}$$

where the deal runs for M months. Solving for the fair contract price k, we get

$$k=\frac{\sum_{m=1}^{M}E\left[P_mL_m\right]}{\sum_{m=1}^{M}E\left[L_m\right]}=\frac{\sum_{m=1}^{M}E[P_m]E[L_m]exp(\rho_m(\sigma_P)_m(\sigma_L)_mt_m)}{\sum_{m=1}^{M}E\left[L_m\right]} \tag{2.2}$$

Note that this approach requires the computation of the expected load, load-price correlation and the load volatility, which is done actuarially. This paper explores an alternative approach where the volume can be approximated as a function of price using a call spread function, which allows us to price the swap and compute its Greeks while eliminating the actuarial parameters. Also note that (2.2) accounts for seasonality by effectively evaluating a k_m for each month

FIGURE 1. Fitting a call spread functional form to all historical load and price data

and converting that to a single k for the term.

2.1. **Functional form of load.** The BGE RES load from the MD-SOS deal has been chosen as an example for this study, given the importance of this load to the FR desk and its highly weather-responsive nature. As has been consistently observed by the team, the load and price are positively correlated, but the correlation tends to be suppressed at extremely low and high prices. A call spread function of the following form was hence chosen to model load as a function of price.

$$L(P) = \begin{cases} L_H, & \text{if } P \geq k_H, \\ L_L + \frac{L_H - L_L}{k_H - k_L}(P - k_L), & \text{if } k_L < P < k_H, \\ L_L, & \text{if } P \leq k_L \end{cases} \tag{2.3}$$

(2.3) can be combined into a single equation as follows:

$$L(P) = N_L + lev(max(P - k_L, 0) - max(P - k_H, 0)) \tag{2.4}$$

where $lev = \frac{L_H - L_L}{k_H - k_L}$. The arbitrage-free strike k_m for a given month can then be solved for as

$$k_m = \frac{E\left[P_m L_m\right]}{E\left[L_m\right]} \tag{2.5}$$

Note that we calculate the k_m for a given month, given that electricity forwards and options trade for a given month. k_m can then be solved as

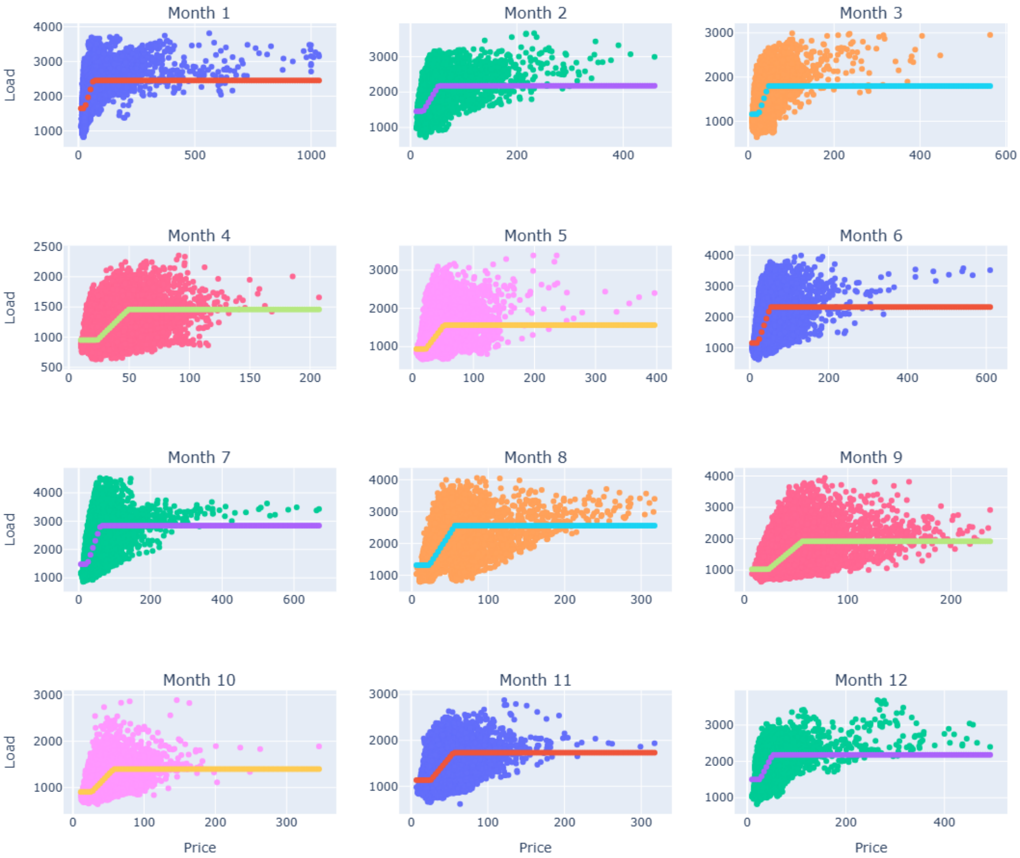

FIGURE 2. Fitting a call spread to data monthly to capture seasonality

$$k_m = F_m \frac{(L_L)_m + lev_m CS(\widetilde{F_m})}{(L_L)_m + lev_m CS(F_m)} \tag{2.6}$$

where F_m is the forward for month m and $\widetilde{F_m} = F_m exp(\sigma^2 t_m)$, where σ_m is the implied volatility for month m and t_m is the time from the evaluation date to the middle of the month in question. CS is the call spread which can be evaluated using the Black-Scholes equation.

The call spreads in 1 and 2 were fitted using the SQSLP (Sequential Least SQuares Programming) optimizer method within the scipy.optimize module, to calculate the optimal parameters k_L, k_H, L_L, L_H for each month, which are then plugged into (2.6) to calculate the k_m for each month. These are then compared to that obtained from FR's MM (MasterModel) as seen in 3. As observed, the results are reasonably similar, with differences averaging to approximately 6%.

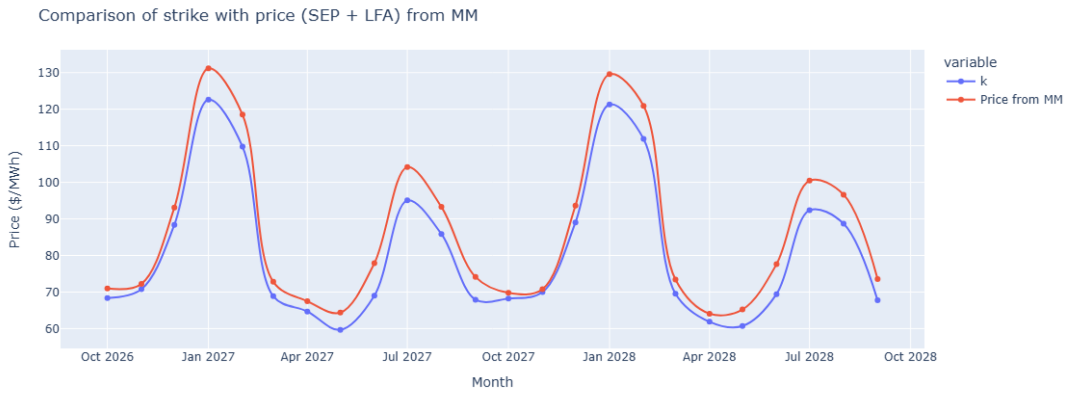

Figure 3. Comparison of calculated k_m with MM results

3. Greeks and Hedging

The delta Δ_m of the swap for a given month can be calculated as

$$\Delta_m = k_m.lev_m.\Delta(CS(F_m)) - \Big((L_L)_m + lev_m.CS(\widetilde{F_m}) + F_m.lev_m.\Delta(CS(\widetilde{F_m})).exp(\sigma_m^2 t_m)\Big) \tag{3.1}$$

The gamma γ_m of the swap for a given month can be calculated as

$$\gamma_m = k_m.lev_m.\gamma(CS(F_m)) - lev_m exp(\sigma_m^2 t_m)\left(2\Delta(CS(\widetilde{F_m})) + \widetilde{F_m}\gamma(CS(\widetilde{F_m}))\right) \tag{3.2}$$

The vega ν_m of the swap for a given month can be calculated as

$$\nu_m = k_m.lev_m.\nu(CS(F_m)) - F.lev_m\left((2\widetilde{F_m}\sigma_m t_m\Delta(CS(\widetilde{F_m})) + \nu(CS(\widetilde{F_m})))\right) \tag{3.3}$$

The theta θ_m of the swap for a given month can be calculated as

$$\theta_m = k_m.lev_m.\theta(CS(F_m)) - F.lev_m\left((-\widetilde{F_m}\sigma_m^2\Delta(CS(\widetilde{F_m})) + \theta(CS(\widetilde{F_m})))\right) \tag{3.4}$$

The Δ neutral hedge volumes calculated using (3.1), compared to the modified volumes (volume such that the derivative can be booked as a regular swap) and FR's hedge volumes can be seen in 4. It is evident that the FR hedge volumes are

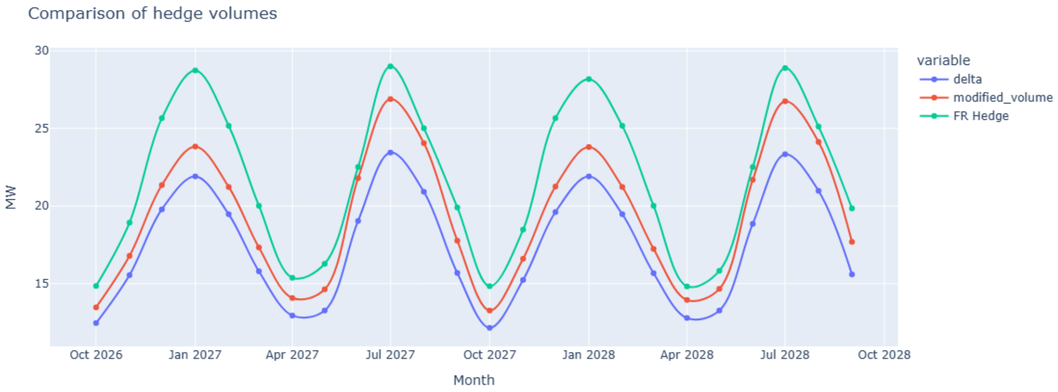

Figure 4. Comparison to hedge volumes from LS

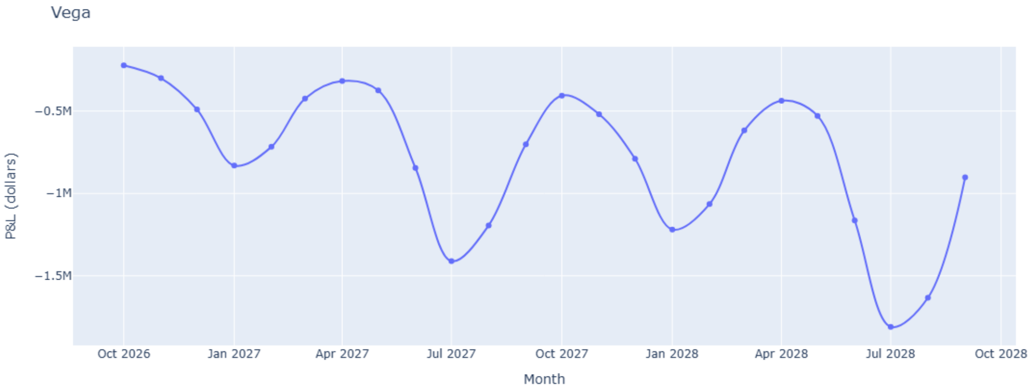

Figure 5. Vega - change in P&L for 1% rise in market volatility

calculated using actuarial overhedge percentages applied to the expected volume. The Δ calculated using (3.1) changes dynamically depending on the forward and the implied volatility and would require rebalancing of the portfolio.

The short vega ν and γ position of the FR portfolio can be seen in 5 and 7. The vega estimate of the overall portfolio by extrapolating this deal is around \$19M for 1% change in volatility, while the gamma estimate of the overall portfolio is around 6 MW for 1% change in the forward. The long θ position in 6 indicates that the overall portfolio makes around \$1.2M per day holding other market variables constant.

3.1. **Conclusion.** Coming up with a fixed price in a variable volume swap (volume can be anything from 0 to ∞) involves using historical data. One can also solve this problem using the principles of financial derivatives by describing the volume as a piecewise linear function of the price, defined by parameters. Optimizing these parameters will result in a "mean" functional form. AI can help

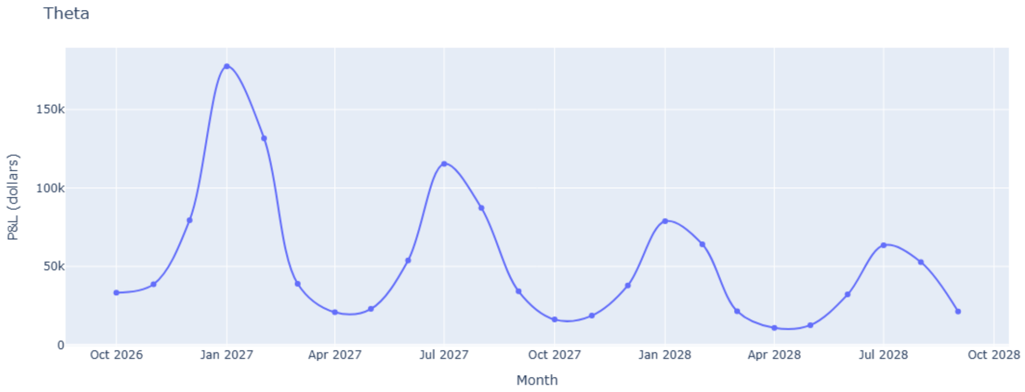

Figure 6. Theta - change in P&L per day

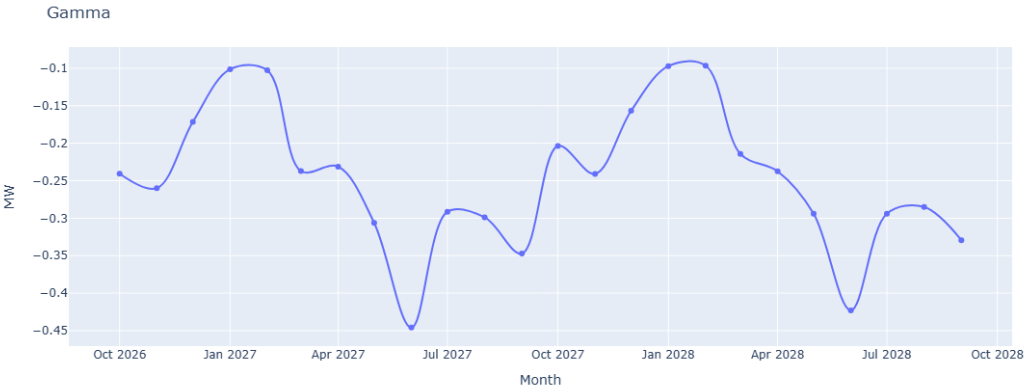

Figure 7. Gamma - change in Delta for 1% rise in forward

come up with a more (potentially) accurate and predictive form using current data, such as demand/supply and weather forecasts.

References

1. Hull, John, 1946-. Options, Futures, and Other Derivatives. Boston: Prentice Hall, 2012.
2. Pricing and Hedging Variable Volume Swaps - `K:/Valuation/Analysts/Hemanth/Exploring LFA/Variable_volume_swap_valuation.pdf`
3. Code (section: Volume as a function of price) - `K:/Valuation/Analysts/Hemanth/Python Notebooks/Miscellaneous/Geometric_Brownian_Motion.ipynb`